

## Chapter 10

### The Economics of Antitrust Law

#### Cartels

Consider the case of  $n$  firms sharing a market. Let

$$\begin{aligned}q_i &= \text{output of firm } i; \\Q &= \sum_{i=1}^n q_i = \text{total industry output}; \\P(Q) &= \text{inverse demand function} \\C(q) &= \text{cost of production for each firm.}\end{aligned}$$

The profit for each firm is given by

$$\pi_i = P(Q)q_i - C(q_i). \quad (10.1)$$

If the firms were able to collude and act like a monopolist, they would jointly solve the following problem:

$$\max_{q_1, \dots, q_n} \sum_{i=1}^n \pi_i = P(Q)Q - \sum_{i=1}^n C(q_i) \quad (10.2)$$

The first order condition for each  $q_i$  is

$$P + P'Q = C', \quad (10.3)$$

which says that the common marginal revenue equals each firm's marginal cost. As an example, let  $P(Q) = a - bQ$  (linear demand) and  $C(q) = cq$  (constant marginal cost). In that case, (10.3) becomes

$$a - bQ - bQ = c, \quad (10.4)$$

which yields optimal aggregate output of

$$Q^* = \frac{a - c}{2b}. \quad (10.5)$$

(We thus need to assume that  $a > c$  so that optimal output is positive.) The corresponding price is  $P(Q^*) = P^*$ , or

$$P^* = \frac{a + c}{2}. \quad (10.6)$$

Note that there is no determinate output level for each firm in this model (due to the constant marginal costs), but if they each produce an equal share of the total,

$$q^* = Q^*/n = \frac{a - c}{2nb}. \quad (10.7)$$

The instability of a cartel can be seen by noting that at the optimum,  $P^* > MC = c$  for each firm. (This is true by (10.6), given the assumption that  $a > c$ .) Thus, each firm, taking the price as given, would like to increase its output above the level in (10.7) until  $P = MC$ . However, the resulting increase in output will cause the market price to fall, thereby eroding the cartel's profits.

*The Cournot model.* It is interesting to compare the above cartel model to the Cournot equilibrium. In this model, each firm maximizes its profit in (10.1), taking the output of all other firms as given. The resulting first order condition for each firm  $i$  is

$$a - bQ - bq_i = c,$$

or

$$a - 2bq_i - b \sum_{j \neq i} q_j = c. \quad (10.8)$$

Solving for  $q_i$  yields

$$q_i = \frac{a - c - b \sum_{j \neq i} q_j}{2b}. \quad (10.9)$$

In a symmetric equilibrium,  $q_i = q_j = q$ . Thus, in the numerator of (10.9),  $\sum_{j \neq i} q_j = (n - 1)q$ . Using this, we can solve (10.9) for the Cournot equilibrium output per firm:

$$q^c = \frac{a - c}{b(n + 1)}. \quad (10.10)$$

The resulting aggregate level of output is

$$Q^c = nq^c = \frac{n(a - c)}{b(n + 1)}, \quad (10.11)$$

and the equilibrium price is

$$P^c = \frac{a + nc}{n + 1}. \quad (10.12)$$

Now observe that if  $n=1$ , (10.11) and (10.12) reduce to (10.5) and (10.6), respectively. At the other extreme, if we take the limit as  $n \rightarrow \infty$ , (10.11) and (10.12) reduce to<sup>1</sup>

$$Q = \frac{a - c}{b} \quad (10.13)$$

and

$$P = c, \quad (10.14)$$

which represent the competitive outcome. Thus, the Cournot model contains both the monopoly (cartel) outcome, and the competitive outcome, as special cases.

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<sup>1</sup> Taking the limit in these expressions requires use of l'Hopital's rule.